University of the Saarland
Department 6.2 - Informatik
Prof. Dr. W.J. Paul

Computer Architecture III - WS 03

(due: 01/30/2004)

Excercise 1: (Probability once again) **(2 points)**

Show that for a random, r–wise independent, uniform hash function h with n–many hash addresses, r distinct values $y_1, y_2, \ldots, y_r$ and r distinct values $x_1, x_2, \ldots, x_r$ holds:

$$Prob\,(h(x_1) = y_1 \wedge \cdots \wedge h(x_r) = y_r) = \frac{1}{(n)^r}.$$

Excercise 2: (Algebra) **(3 points)**

Show: For r distinct values $y_1, y_2, \ldots, y_r$ and r distinct values $x_1, x_2, \ldots, x_r$ it holds: There exists at most 1 function f with the following property:

$$f(x) = \sum_{i=0}^{r-1} \beta_i x^i \quad s.t.\ \forall i : f(x_i) = y_i$$

Hint: Use the "Van der Waerden" Theorem from the lecture.

If we don't reach Ranade Routing in the lecture on 01/23/04 then the next two exercises are postponed by one week.

Excercise 3: (Ranade Routing) **(3 points)**

Remember from the lecture that the probability that a packet is delayed by Δ steps is at most $\Delta n^4 \left(\frac{2^9 Ce}{\Delta}\right)^{\frac{\Delta}{2}}$. Show that this probability gets extremely small for $C \leq \frac{log\,n}{2}$ and $\Delta = \frac{2^{10} e\, log\, n}{log(\frac{log\, n}{C})}$.

Excercise 4: (Ranade Routing) **(5 points)**

In the lecture we used very big constant factors in Δ. Show how to dramatically reduce these constants for a queue size $Q \geq 6$.
Hint: Try to replace the 2^{10} term with something really small (≤ 10).