University of the Saarland
Department 6.2 - Informatik
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Computer Architecture III - WS 03

(due: 12/12/2003)

Excercise 1: (Lower bound proof using Kolmogorov complexity) **(4 + 2 points)**
Let $\omega = \omega_L\omega_R$ with $|\omega_L| = |\omega_R| = \frac{n}{2}$. We want to test $\omega_L = \omega_R$ on a parallel machine whose $n+1$ processors are connected like a tree. ω_L is input on the left subtree, ω_R on the right subtree connected with the root. We assume that all processors run the same program.

Show:

1. for some constant $c > 0$ at least $c \cdot n$ steps are necessary to determine wether $\omega_L = \omega_R$.
2. Assume every processor P_i runs a different program u_i. Where does the proof of part 1) collapse?

Excercise 2: (Relative Kolmogorov complexity) **(2 + 2 + 2 points)**
We define the Kolmogorov complexity of x given y as the length of the shortest description of x given y as auxiliary data:

$$K(x|y) = min\{|u'v| : M_u \text{ started with } v\#y \text{ prints } x \text{ and halts}\}$$

Let $|u| = n$. Show:

1. $K(\bar{u}|u) = O(1)$,
2. $K(u|v) \leq K(u|y) + K(y|v) + O(logn)$,
3. there exists u such that $K(u|v) \geq n$.

Excercise 3: (Kolmogorov complexity of random bit vectors) **(3 points)**
Let $x = \{0,1\}^n$ be a n-bit bitvector where all bits are randomly choosen using an uniform distribution. Proof that

$$p(K(x) \geq n - c) = 1 - 2^c.$$

Excercise 4: (Probability) **(1 point)**
Let A and B be events. Show that $p(A \cup B) = p(A) + p(B) - p(A \cap B)$.