University of the Saarland
Department 6.2 - Informatik
Prof. Dr. W.J. Paul

Computer Architecture III - WS 03

(due: 12/05/2003)

Excercise 1: (Marriage Theorem) **(4 points)**

In the lecture we proved that the premise of the marriage theorem ($\forall V \subset V_1 : \#V \leq \#E(V)$) is *sufficient* to proof the marriage theorem. Proof or disprove that it is also *neccessary*, i.e. that the theorem is wrong without this premise.

Excercise 2: (Probability) **(2 + 1 points)**

Let (Ω_1, pr_1) and (Ω_2, pr_2) be two discrete probability spaces. Let (Ω, pr) be defined as $\Omega = \Omega_1 \times \Omega_2$ and (for $\omega_1 \in \Omega_1$ and $\omega_2 \in \Omega_2$) $pr(\omega_1, \omega_2) = pr_1(\omega_1) \cdot pr_2(\omega_2)$.

- Show that for events $A \subset \Omega_1$, $B \subset \Omega_2$ and $A' = A \times \Omega_2$, $B' = \Omega_1 \times B$ it holds that A' and B' are independent.
- Proof that $pr(A \times B) = pr_1(A) \cdot pr_2(B)$.

Excercise 3: (Binomial Coefficients) **(3 points)**

Proof the following Lemma:

$$0 < y < x \Longrightarrow \binom{x}{y} < \left(\frac{x \cdot e}{y}\right)^y .$$

Hint: In your proof you can assume that $n! \geq e \cdot \left(\frac{n}{e}\right)^n$.