



Exercise 1: (Significand Normalization Shifter)

(10 Points)

Figure 1 depicts the implementation of the significand normalization shifter in the FPU rounding unit. The significand $f' = f_r/2 > 0$ is put in a cyclical left shifter with shift amount $\langle sh[5 : 0] \rangle$ where $\sigma = \langle sh[12 : 0] \rangle \leq 56$ is the desired left shift distance. Additionally masks $v, w \in \mathbb{B}^{64}$ are computed and anded to the shifter output fs to obtain the normalized significand. In the analysis of the MASK circuit we already derived that $v = \bar{u}$ and $\forall i \in [0 : 63]. w_i = u_i \wedge sh[12]$ with:

$$u[0 : 63] = \begin{cases} 0^{64-\sigma} 1^\sigma & : 0 \leq \sigma \\ 1^{|\sigma|} 0^{64-|\sigma|} & : -63 \leq \sigma \leq -1 \\ 1^{64} & : \sigma < -63 \end{cases}$$

Let $f_n = \langle fn[0].fn[1 : 127] \rangle$. To be proven:

$$f_n =_p 2^\sigma \cdot f'$$

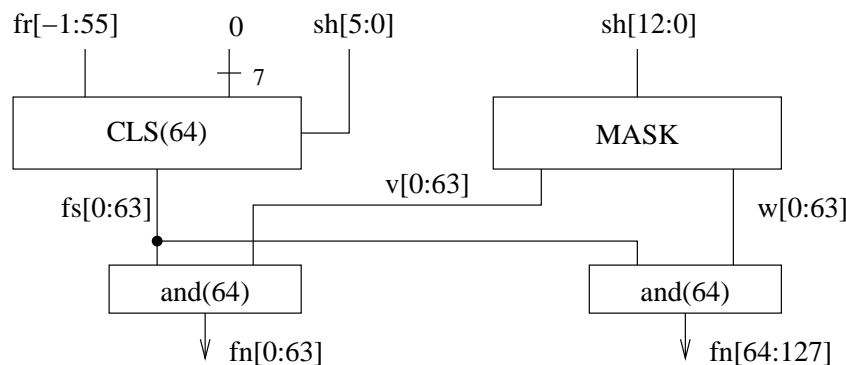


Figure 1: Significand Normalization Shifter

Exercise 2: (Improved Exponent Normalization Shifter)

(6+2 Points)

Figure 2 shows the exponent normalization shifter from the FPU rounding unit. One could speed up this circuit by getting rid of the 3/2 Adder. This is done by combining two of the inputs.

- Show how this can be achieved and that your construction works.
- How does this modification affect the overall cost and delay of the circuit *ExpNorm*?

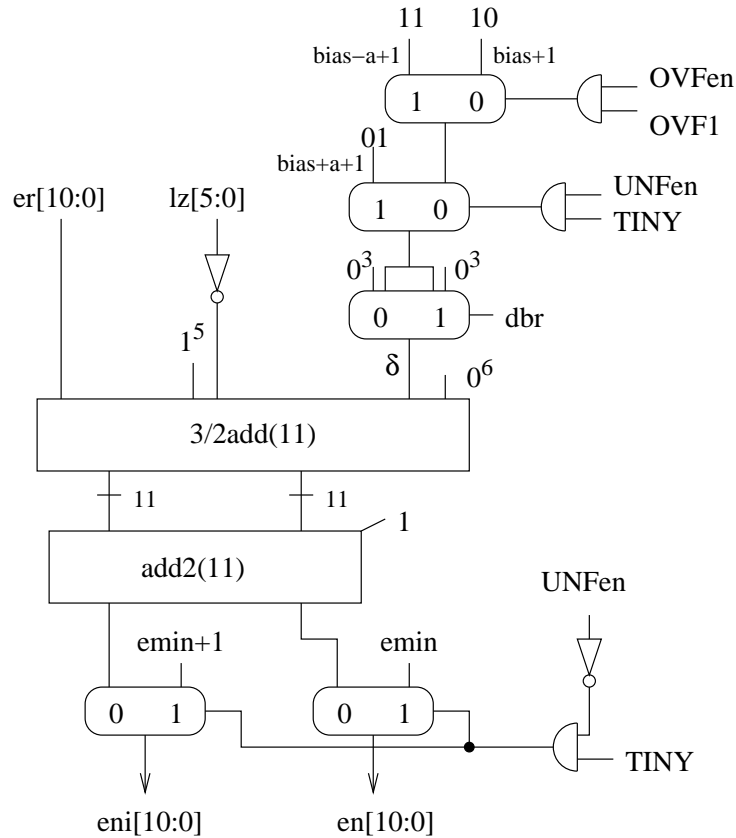


Figure 2: Exponent Normalization Shifter

Exercise 3: (Fast and Cheap Incrementers)

(3+3+3+8+5 Points)

In the significand rounder we need a large (53-bit) incremter. An n -bit incremter is a circuit with inputs $a \in \mathbb{B}^n$, $inc \in \mathbb{B}$ and outputs $s \in \mathbb{B}^n$, $c \in \mathbb{B}$ such that the property

$$\langle c \ s[n-1:0] \rangle = \langle a[n-1:0] \rangle + inc$$

holds. A fast incremter with logarithmic delay and linear-logarithmic cost can be implemented in a way similar to the Conditional Sum Adder.

- Construct an n -bit Conditional Sum Incrementer for $n = 2^k$!
- Prove the correctness of your construction!
- How would you optimize your construction for $n \neq 2^k$?
- Give the exact cost and delay of your Incrementer for $n = 2^k$ in non-recursive formulas! Prove their correctness!
- Consider a Carry Lookahead implementation of an incrementer! How does it compare to the Conditional Sum Incrementer concerning cost and delay?