Universität des Saarlandes
FR 6.2 - Informatik
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Exercise Sheet 3
Computer Architecture II
(Due: Nov 12th, 2013)

Note: If you want to take the exam you have to register in HISPOS until Nov 17th!

Exercise 1: (Representable Numbers) **(3+5 Points)**

Using IEEE floating-point numbers only a finite subset $\mathcal{R}$ of the real numbers can be represented. With respect to some precision α, all numbers not being representable are associated with corresponding α-representatives. In order to obtain the floating-point representation these representatives are converted into a factoring, normalized and rounded according to one of the four IEEE rounding modes.

a) Determine the binary fractional representation for $\left[\frac{1}{3}\right]_\alpha$! Distinguish the cases of even and odd α!

b) Find the IEEE floating-point representation $(s, e[n-1:0], f'[1:p-1])$ such that:

$$[\![s, e, f']\!] = r_{ne}\left(\frac{1}{3}\right)$$

Consider both single and double precision! Are the numbers even or odd?

Exercise 2: (Properties of α-Equivalence) **(3+3+2+6 Points)**

Let $x, x' \in \mathbb{R}$ and $\alpha, q \in \mathbb{Z}$. Assume $x =_\alpha x'$ and prove the following Lemmas:

a) $-x =_\alpha -x'$ and $[-x]_\alpha = -[x]_\alpha$ (Mirroring)

b) $2^e \cdot x =_{\alpha-e} 2^e \cdot x'$ and $[2^e \cdot x]_{\alpha-e} = 2^e \cdot [x]_\alpha$ (Scaling)

c) Let $y = q \cdot 2^{-\alpha}$ then $x + y =_\alpha x' + y$ (Translation)

d) Let $\beta < \alpha$ then $x =_\beta x'$ (Refinement)

Exercise 3: (Rounding with Unlimited Exponent Range) **(4+2+4 Points)**

Let $x \in \mathbb{R} \setminus \{0\}$, $\hat{\eta}(x) = (s, \hat{e}, \hat{f})$, and $\hat{r} : \mathbb{R} \to \hat{\mathcal{R}}$ be an IEEE rounding mode for rounding with unlimited exponent range. Prove that:

a) $\hat{r}(x) = \hat{r}([x]_{p-\hat{e}})$

b) If $x =_{p-\hat{e}} x'$ then $\hat{r}(x) = \hat{r}(x')$

c) $\hat{\eta}([x]_{p-\hat{e}}) = (s, \hat{e}, [\hat{f}]_p)$

Exercise 4: (Significand Rounding) **(4+4 Points)**

Assume a real number x with an IEEE-normal factoring (s, e, f) such that $s = 1,\ f \in [1, 2)$ and $x = [\![s, e, f]\!]$. Let $x_1 = [\![s, e, f_1]\!]$ with $f_1 = sigrd_u(s, f)$. Show for

a) $|x| \leq X_{max}$

b) $|x| > X_{max}$

that the significand rounding works correctly:

$$x_1 = \begin{cases} r(x) & : \quad |x| \leq X_{max} \\ \hat{r}(x) & : \quad |x| > X_{max} \end{cases}$$