Universität des Saarlandes
FR 6.2 - Informatik
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Exercise Sheet 1
Computer Architecture II

(Due: Nov 3rd, 2011)

Organisational Information:

- Each week on Tuesday one exercise sheet will be released. The solutions should be handed in before or after the next Tuesday lecture if not stated otherwise. For the admission to the exam you will need at least 50% of the points of the exercises.
- The tutorial takes place on Thursdays, 16:00 to 18:00 in E1.7, SR 323 starting from Oct 27th.
- You are allowed to solve the exercise sheets in groups of two people. Everybody who has his name on the solution must be able to present it in the tutorials. Everybody must present the solution of at least two excercises.
- Please, register for the lecture at the lecture's webpage until Nov 8th, 2011!
  `http://www-wjp.cs.uni-saarland.de/lehre/vorlesung/rechnerarchitektur2/ws1112/`
  Also do not forget to register for the exam in the HISPOS system!
- The oral exam will take place in February. An exact date will be decided upon in class.

Exercise 1: (Decomposition Lemma) **(5 Points)**

Prove the following Lemma for an n-digit number representation $a \in \{0, \ldots, B-1\}^n$ with $B \in \mathbb{N}^+$, $m, n \in \mathbb{N}^+$, $n > 1$ and $m < n$!

$$\langle a[n-1:0]\rangle_B \quad = \quad \langle a[n-1:m]\rangle_B \cdot B^m + \langle a[m-1:0]\rangle_B$$

Note: With $\langle a\rangle_B$ we denote the value of the number representation a regarding the base B.

$$\langle a[n-1:0]\rangle_B \equiv \sum_{i=0}^{B-1} a_i \cdot B^i$$

Also we agree on the convention $\langle a[n-1:0]\rangle \equiv \langle a[n-1:0]\rangle_2$.

Exercise 2: (Two's Complement Numbers) **(2+1+2+2+3 Points)**

In class, two's complement numbers and their definition were presented. In this exercise you have to show that the following properties hold for $a \in \mathbb{B}^n$.

a) $[a] < 0 \iff a_{n-1} = 1$

b) $[0a] \equiv \langle a\rangle$

c) $[a] \equiv \langle a\rangle \mod 2^n$

d) $[a_{n-1}a] \equiv [a]$

e) $-[a] \equiv [\overline{a}] + 1$

Exercise 3: (Representable Numbers) **(3+5 Points)**

Using IEEE floating-point numbers only a finite subset $\mathcal{R}$ of the real numbers can be represented. With respect to some precision α, all numbers not being representable are associated with corresponding α-representatives. In order to obtain the floating-point representation these representatives are converted into a factoring, normalized and rounded according to one of the four IEEE rounding modes.

a) Determine the binary fractional representation for $\left[\frac{1}{3}\right]_\alpha$! Distinguish the cases of even and odd α!

b) Find the IEEE floating-point representation $(s, e[n-1:0], f'[1:p-1])$ such that:

$$[\![s, e, f']\!] = r_{ne}\left(\frac{1}{3}\right)$$

Consider both single and double precision! Are the numbers even or odd?

Exercise 4: (Properties of α-Equivalence) **(3+3+2+6 Points)**

Let $x, x' \in \mathbb{R}$ and $\alpha, q \in \mathbb{Z}$. Assume $x =_\alpha x'$ and prove the following Lemmas:

a) $-x =_\alpha -x'$ and $[-x]_\alpha = -[x]_\alpha$ (Mirroring)

b) $2^e \cdot x =_{\alpha-e} 2^e \cdot x'$ and $[2^e \cdot x]_{\alpha-e} = 2^e \cdot [x]_\alpha$ (Scaling)

c) Let $y = q \cdot 2^{-\alpha}$ then $x + y =_\alpha x' + y$ (Translation)

d) Let $\beta < \alpha$ then $x =_\beta x'$ (Refinement)

Exercise 5: (Rounding with Unlimited Exponent Range) **(4+2+2 Points)**

Let $x \in \mathbb{R} \setminus \{0\}$, $\hat{\eta}(x) = (s, \hat{e}, \hat{f})$, and $\hat{r} : \mathbb{R} \to \mathcal{R}$ be an IEEE rounding mode. Prove that:

a) $\hat{r}(x) = \hat{r}([x]_{p-\hat{e}})$

b) $\hat{\eta}([x]_{p-\hat{e}}) = (s, \hat{e}, [\hat{f}]_p)$

c) If $x =_{p-\hat{e}} x'$ then $\hat{r}(x) = \hat{r}(x')$