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Department 6.2 - Computer Science
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Computer Architecture II - WS 08/09

Exercise Sheet 5 (due: 1.12.08)

Organizational Notes:

- **Students wanting to take part in the examination have to register for it (HISPOS). The HISPOS-registration for the examination is active from Nov 17th until Dec 05th 2008. Those who have not subscribed to the examination, cannot take part in it or if he/she does, the result cannot be credited.**

Excercise 1: (Table Size vs. Number of Iterations) (6 points)
In the lecture we designed the multiply/divide unit from the book which performs a division based on the Newton-Raphson method. The iteration starts out with an initial approximation x_0 which is obtained from a $2^\gamma \times \gamma$ lookup table. The intermediate results are truncated after $\sigma = 57$ bits. The number i of iterations necessary to reach $p + 2$ bits of precision (i.e. $\delta_i < 2^{-(p+2)}$) is then bounded by

$$i = \begin{cases} 1 & if \ \ (p = 24) \wedge (\gamma = 16) \\ 2 & if \ \ (p = 24) \wedge (\gamma = 8) \vee (p = 53) \wedge (\gamma = 16) \\ 2 & if \ \ (p = 24) \wedge (\gamma = 5) \vee (p = 53) \wedge (\gamma = 8) \end{cases}$$

To be proven: the number of iterations suffices to achieve the desired precision.

Excercise 2: (3/2 Carry Save Adder with Carry-In) (3 points)
Let's consider a 3/2 carry save adder circuit having four inputs: $a, b, c \in \{0,1\}^n$ and c_{in}, and two outputs: $t, s \in \{0,1\}^n$.

$$\langle s \rangle + \langle t \rangle = \langle a \rangle + \langle b \rangle + \langle c \rangle + \langle c_{in} \rangle \, mod \; 2^n$$

In this exercise you need to show that the above presented statement holds. (Hint: $t_0 = c_{in}$)

Excercise 3: Binary Fractions (6 points)
Let $\langle a_n.f_p \rangle = \sum_{i=0}^{n-1} a_i 2^i + \sum_{i=1}^{p-1} f_i 2^{-i}$. Let $x = \langle a'_s.f'_u \rangle$ and $y = \langle a''_t.f''_v \rangle$ be binary fractions. For $s > t$ and $u < v$ prove the following statements:

- $x + y = 2^{-v}(\langle a'_s f'_u 0^{v-u} \rangle + \langle 0^{s-t} a''_t f''_v \rangle)$
- $x * y = 2^{-2v}(\langle a'_s f'_u 0^{v-u} \rangle * \langle 0^{s-t} a''_t f''_v \rangle)$