

Computer Architecture II - WS 08/09
Exercise Sheet 1 (due: 3.11.08)

Organizational Notes:

- The registration for this lecture is opened until **November 10th!**
- Exercise sheets will be handed out on Mondays before the lecture. Your solutions will be collected one week after that (before the lecture).
- You are allowed to solve the exercises in groups of up to 2 students. Groups should not change over the semester. Everybody who has his name on a solution must be able to present it in tutorial.
- You need to solve 50% of all exercises in order to be admitted to the exam. In addition you must successfully present at least four solutions in tutorial.
- There will be one **oral exam** taking place at the end of the semester.

Exercise 1: (full adder) (2 points)

Design a cheapest full adder (single bit adder). The full adder takes three bits (a , b and carry-in c_{in} bits) as input and produces two signals (carry-out c_{out} and sum s bits) as output.

You can use the following basic circuits to construct your full adder: AND, OR, INV, NAND, NOR, XOR, XNOR, MULTIPLEXER. All these basic circuits have cost one. In the class, the full adder was presented and it has cost seven. The cheapest full adder design has cost three.

Exercise 2: (decomposition lemma) (2 points)

Let $a \in \{0, 1\}^n$ and $\langle a \rangle$ its binary representation. Prove that for all $k \in \{0, \dots, n-1\}$ the following equation holds:

$$\langle a[n-1:0] \rangle = \langle a[n-1:k] \rangle \cdot 2^k + \langle a[k-1:0] \rangle$$

Exercise 3: (two's complement numbers) (6 points)

In the class, two's complement numbers and their properties were presented. In this exercise you need to show that these properties hold for $a \in \{0, 1\}^n$:

1. $[0, a] = \langle a \rangle$
2. $[a] \equiv \langle a[n-2:0] \rangle \pmod{2^{n-1}}$
3. $[a] \equiv \langle a \rangle \pmod{2^n}$
4. $[a_{n-1}, a] = [a]$
5. $-[a] = [\bar{a}] + 1$