University of the Saarland
Department 6.2 – Informatik
Prof. Dr. W.J. Paul

Computer Architecture II – WS 05/06

(due: Monday, 14.11.2005)

For the admission to the exam you will need at least 50% of the points of the excercises. You are allowed to solve the exercise sheets in groups of three people. But everybody who has his name on the solution must be able to present it in the tutorials. Everybody must present the solution of at least four excercises.

Exercise 1: (Computation Rules) **(2 + 2 + 2 + 2 + 2 + 2 points)**
In the lecture we introduced some computation rules for α-equivalence and α-representatives. Let $x =_\alpha x'$. To be proven:

1. $-x =_\alpha -x'$
2. $[-x]_\alpha = -[x]_\alpha$
3. $2 \cdot x =_{\alpha-1} 2 \cdot x'$
4. $2^e \cdot x =_{\alpha-e} 2^e \cdot x'$
5. Let y be a multiple of $2^{-\alpha}$. Then it holds: $x + y =_\alpha x' + y$
6. Let $\beta < \alpha$ then $x =_\beta x'$

Exercise 2: (Rounding Theory) **(15 points)**
Rounding with unlimited exponent. In the lecture we defined the rounding function $\hat{r}_m$ for an $m \in \{u, d, z, ne\}$.

$$\hat{r}_m : \mathbb{R} \to \mathcal{R}$$

Let $x \neq 0$, $\hat{\eta}(x) = (s, \hat{e}, \hat{f})$. To be proven:

1. $\hat{r}_m(x) = \hat{r}_m([x]_{p-\hat{e}})$
2. $\hat{\eta}([x]_{p-\hat{e}}) = (s, \hat{e}, [\hat{f}]_p)$
3. $x' =_{p-\hat{e}} x \Rightarrow \hat{r}_m(x') = \hat{r}_m(x)$

Exercise 3: (Sticky Bit Computation) **(8 points)**
Let $f = f[-u:0].f[1:v]$ be a binary fraction. Given a p with $1 < p < v$ let:

$$g = f[-u:0].f[1:p]$$

and let

$$s = \bigvee_{i=p+1}^{v} f[i]$$

be the sticky bit of f for position p.

To be proven:

$$[\langle f \rangle]_p = \langle gs \rangle$$

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Exercise 4: (Binary Fractions) **(5 points)**

Let x, y be 2's complement fractions with $s > t$ and $u < v$:

- $\left[a[n-1:0].f[1:p-1]\right] = -a[n-1] \cdot 2^{n-1} + \langle a[n-2:0].f[1:p-1]\rangle$
- $x = \left[a[s-1:0].f[1:u]\right]$
- $y = \left[b[t-1:0].g[1:v]\right]$

To be proven:

$$x + y = (\left[af0^{v-u}\right] + \left[b_{t-1}^{s-t}b[t-1:0]g\right]) \cdot 2^{-v}$$