University of the Saarland
Department 6.2 - Informatik
Prof. Dr. W.J. Paul

Computer Architecture II - WS 05/06

(due: Monday, 31.10.2005)

Excercise 1: (Number Representation) **(10 points)**

In the lecture we defined the binary representation of numbers. We will now extend that definition.

$$B, h \in N;\ B \geq 2;\ 1 \leq h \leq (n-1)$$

Def:

$$\langle a[n-1:0]\rangle_B \ = \sum_{i=0}^{n-1} a_i \cdot B^i$$

To be proven:

$$\langle a[n-1:0]\rangle_B \ = \ \langle a[n-1:h]\rangle_B \cdot \ B^h + \langle a[h-1:0]\rangle_B$$

Excercise 2: (Recursive Estimations Warmup) **(5 points)**

In the lecture we treated the construction of *conditional sum adders*. The following information was given:

$$D(1) = D(FA)$$
$$D(n) = D(Mux) + D(\frac{n}{2})$$

Assume that n is a power of two ($n = 2^k$). To be proven:

$$D(n) = \log_2 n \cdot D(Mux) + D(FA)$$

Excercise 3: (Recursive Estimations Continued) **(10 points)**

In the lecture we treated the construction of a *compound adder*. We can estimate the cost of the construction using:

$$C(1) \leq 2 \cdot C(FA)$$
$$C(n) \leq \ C(\frac{n}{2}) + (n+2) \cdot C(Mux)$$

Assume that n is a power of two ($n = 2^k$). To be proven:

$$C(n) \in \mathcal{O}(n \log_2 n)$$

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Excercise 4: (*Conditional Sum Adder* Correctness) **(15 points)**

Prove the correctness of the *conditional sum adder* given in the lecture. You can assume the correctness of a *full adder*:

$$x, y, z, c \in \{0, 1\},\ a, b, s \in \{0, 1\}^n$$
$$FA \text{ implements: } \langle c_{out}\,,\, z\rangle \;=\; \langle x\rangle + \langle y\rangle +\; c_{in}$$

Assume that n is a power of two ($n = 2^k$). To be proven:

$$COS_n \text{ implements: } \langle a\rangle + \langle b\rangle +\; c_{in} =\; \langle c_n\,,\, s[n-1:0]\rangle$$