Saarland University
Department 6.2 – Computer Science
Prof. Dr. W. J. Paul
M. Sc. Petro Lutsyk

Computer Architecture – WS14/15

Exercise Sheet 1 (due: 04.11.12, 32 points)

Exercise 1: (congruence mod k) (2 points)

For integers $a, b \in \mathbb{Z}$ and natural numbers $k \in \mathbb{N}$ and defines a and b to be *congruent* mod k iff they differ by an integer multiple of k.

$$a \equiv b \bmod k \leftrightarrow \exists z \in Z : a - b = z \cdot k$$

Prove that congruence mod k is an equivalence relation.

Exercise 2: (decomposition lemma) (2 points)

Let $a \in \mathbb{B}^n$. Prove that for $k \in [0 : n-1]$ the following lemma holds:

$$\langle a[n-1:0]\rangle = \langle a[n-1:k]\rangle \cdot 2^k + \langle a[k-1:0]\rangle$$

Exercise 3: (induction principle) (6 points)

Prove by induction on n:

(a) $\sum\limits_{i=0}^{n} i = n \cdot (n+1)/2$

(b) $\sum\limits_{i=0}^{n-1} q^i = (q^n - 1)/(q-1)$

(c) $\sum\limits_{i=0}^{n} i^3 = (\sum\limits_{i=0}^{n} i)^2$

Exercise 4: (addition algorithm) (4 points)

Prove that addition algorithm works for addition of n-bit numbers: For $a, b \in \mathbb{B}^n$ and $c_0 \in \mathbb{B}$ show that the outputs $s \in \mathbb{B}^n$ and $c_n \in \mathbb{B}$ of the addition algorithm satisfy:

$$\langle c_n, s\rangle = \langle a\rangle + \langle b\rangle + c_0$$

Exercise 5: (adder construction) (4 points)

Prove that construction from Fig. 1 implements the addition algorithm.

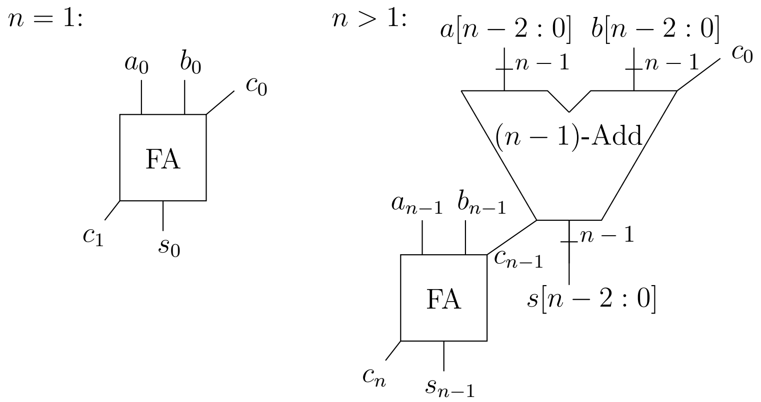

Figure 1: Recursive construction of a carry-chain adder

Exercise 6: (multiplexer construction) (2 points)
Using gates presented in the lecture ($\neg$, $\wedge$, $\vee$ and $\oplus$) give a construction of the circuit which fulfills the following specification:

- inputs: $a, b \in \mathbb{B}^n$ and $s \in \mathbb{B}$
- output: $c \in \mathbb{B}^n$, s.t.

$$c = \begin{cases} a & s = 1 \\ b & s = 0 \end{cases}$$

Exercise 7: (two's complement numbers) (8 points)
Prove the following properties of two's complement numbers:

(a) $[0a] = \langle a \rangle$

(b) $a \in \mathbb{B}^n \rightarrow [a_{n-1}a] = [a]$

(c) $-[a] = [\overline{a}] + 1$

(d) $[a] \equiv \langle a \rangle \bmod 2^n$

Exercise 8: (arithmetic unit) (4 points)
Recall the construction of the AU (see Fig. 2) from the lecture. Derive a recipe for the negative bit computation such that it satisfies

$$neg \leftrightarrow S < 0,$$

where S is the exact result, for

(a) two's complement numbers ($u = 0$)

(b) binary numbers ($u = 1$)

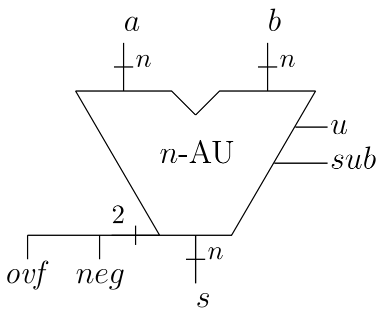

Figure 2: Symbol of an n arithmetic unit (AU)