Saarland University
Department 6.2 - Computer Science
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Computer Architecture - WS12/13

Exercise Sheet 1 (due: 25.10.12, 18 points)

Organizational notes:

- Please, register for the lecture on our website.
- Exercise sheets will be handed out on Thursday before the lecture. Your solutions will be collected one week after that (by the end of the lecture).
- You are allowed to solve the exercises in groups of up to 3 students. Groups should not change over the semester. Everybody who has his name on a solution must be able to present it in tutorial.
- You need to solve 50% of all exercises in order to be admitted to the exam.
- There will be an oral exam taking place at the end of the semester.

Exercise 1: (decomposition lemma) (2 points)

Let $a \in \mathbb{B}^n$. Prove that for all $k \in \{0, \ldots, n-1\}$ the following equation holds:

$$\langle a[n-1:0] \rangle = \langle a[n-1:k] \rangle \times 2^k + \langle a[k-1:0] \rangle$$

Exercise 2: (Congruence *mod k*) (2 points)

For integers $a, b \in \mathbb{Z}$ and natural numbers $k \in \mathbb{N}$ one defines a and b to be *congruent mod k* iff they differ by an integer multiple of k.

$$a \equiv b \bmod k \leftrightarrow \exists z \in \mathbb{Z} : a - b = z \cdot k$$

Prove, that congruence *mod k* is an equivalence relation.

Exercise 3: (*mod* and *tmod* calculations) (3 + 3 points)

Let k be even and $x \equiv y \bmod k$. Prove the following properties:

1. $x \in [0 : k-1] \rightarrow x = (y \bmod k)$
2. $x \in [-k/2 : k/2 - 1] \rightarrow x = (y \; tmod \; k)$

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Exercise 4: (binary representation bijective) (1 + 1 + 1 + 1 points)

In the lecture we defined interpretation of a string as a natural number in the following way:

$$\langle \cdot \rangle \in \mathbb{B}^n \mapsto [0 : 2^n - 1]$$
$$\langle a \rangle = \sum_{i=0}^{n-1} a_i \cdot 2^i$$

Prove, that binary representation of length n is bijective:

1. $\forall a \in \mathbb{B}^n : \sum_{i=0}^{n-1} a_i \cdot 2^i \in [0 : 2^n - 1]$
2. $\forall a, b \in \mathbb{B}^n : a \neq b \rightarrow \langle a \rangle \neq \langle b \rangle$
3. $\forall a, b \in \mathbb{B}^n : \langle a \rangle \neq \langle b \rangle \rightarrow a \neq b$
4. $\forall k \in [0 : 2^n - 1] : \exists a \in \mathbb{B}^n : \langle a \rangle = k$

Exercise 5: (properties of two's complement numbers) (1 + 1 + 1 + 1 points)

Show that the following properties of two's complement numbers hold. Let $a \in \mathbb{B}^n$, then

1. $[0a] = \langle a \rangle$
2. $[a] \equiv \langle a[n-2:0] \rangle \bmod 2^{n-1}$
3. $[a] \equiv \langle a \rangle \bmod 2^n$
4. $[a_{n-1}a] = [a]$