Saarland University
Department 6.2 - Computer Science
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Computer Architecture I - WS 07/08

Exercise Sheet 4

Excercise 1: (warm up)
Show that the binary relation $(=_{\text{mod } k}) : \mathbb{Z}^2 \to \mathbb{B}$ is symmetric, reflexive and transitive.

Excercise 2: (two's complement numbers)
Let $a \in \mathbb{B}^n$. Prove:

1. $[0, a] = \langle a \rangle$
2. $[a] \equiv \langle a[n-2:0] \rangle \mod 2^{n-1}$
3. $[a] \equiv \langle a \rangle \mod 2^n$
4. $[a_{n-1}, a] = [a]$
5. $-[a] = [\neg a] + 1$

Excercise 3: (shifter)

1. Prove the correctness of the n-bit cyclic left shifter:

$$\forall i : r_i = cls(a, \langle b[i:0] \rangle)$$

2. Prove the following shift properties:
   (a) $\langle lls(a,i) \rangle \equiv \langle a \rangle \cdot 2^i \mod 2^n$
   (b) $\langle lrs(a,i) \rangle \equiv \lfloor \langle a \rangle / 2^i \rfloor$
   (c) $[ars(a,i)] \equiv \lfloor [a] / 2^i \rfloor$

Excercise 4: (binary comparision)
Within the ALU construction in the lecture notes, the binary comparison for two two's complement values $[a]$ and $[b]$ with $a, b \in \mathbb{B}^n$ defined as

$$[a] \circ [b] \text{ for } \circ \in \{<, >, \leq, \geq, \neq, =\}$$

is implemented by a subtractor and an equality tester. In this exercise you are asked to construct a different, special circuit for this binary comparison applied to two binary values. Hence, you should construct a circuit that calculates

$$\langle a \rangle \circ \langle b \rangle \text{ for } \circ \in \{<, >, \leq, \geq, \neq, =\}$$

Since your construction should be a special purpose circuit, do not use the construction from the lecture notes in the following.

1. Construct a circuit that computes the output $c \in \mathbb{B}$ with $c = \langle a \rangle < \langle b \rangle$ from the inputs $a, b \in \mathbb{B}^n$.
2. Extend your construction from 1. to compute the six comparisons from above.
3. Compute the delay and cost of your construction as a closed formula and compare those with the construction in the lecture.

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Excercise 5: (half decoder)
An n-half decoder is a circuit with inputs $x[n-1:0]$ and outputs $Y[2^n-1:0]$ such that

$$Y[2^n-1:0] = 0^{2^n-\langle x\rangle}1^{\langle x\rangle}$$

Construct it and prove the correctness of your construction.