University of the Saarland
Department 6.2 - Informatik
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Computer Architecture I - WS 03/04

(due: 19.11.2003)

Exercise 1: (binary representation e_0 of E_0 in Booth recoding) **(4 points)**
To avoid summing of negative numbers C_{2j} in Booth recoding we consider positive E_{2j} instead:

$$\begin{aligned} E_{2j} &= C_{2j} + 3 \cdot 2^{n+1} \\ E_0 &= C_0 + 4 \cdot 2^{n+1} \end{aligned}$$

where corresponding binary representations are:

$$\begin{aligned} e_{2j} &= bin_{n+3}(E_{2j}) \\ e_0 &= bin_{n+4}(E_0) \end{aligned}$$

In the class was proved that binary representation e_{2j} of E_{2j} can be computed by

$$\langle e_{2j} \rangle = \langle 1\overline{s_{2j}}, d_{2j} \oplus s_{2j} \rangle + s_{2j} \text{ for } j > 0$$

Derive the formula of binary representation for case $j = 0$ and prove its correctness.

Exercise 2: (the last sign bit) **(8 points)**
Constructing partial products for multiplication using Booth Recoding we incorporated sign bits s_{2j} into the representation of F_{2j+2} instead of adding them to F_{2j} and defined

$$\begin{aligned} g_{2j} &= (f_{2j} 0 s_{2j-2}) \\ g_0 &= (f_0 00) \end{aligned}$$

Prove that $s_{2m'-2} = 0$, i.e. we don't need to perform the additional summation in order to add the last sign bit (see Figure 1).

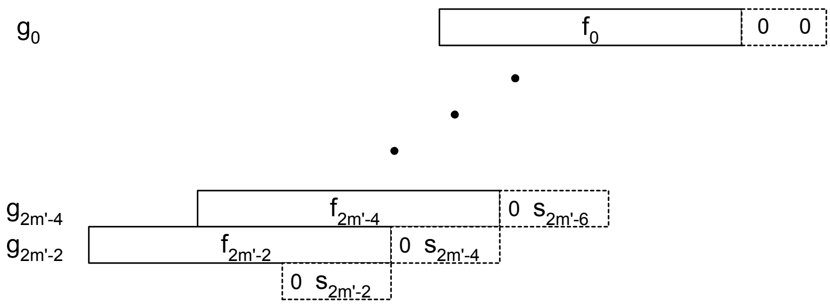

Figure 1: Construction of the partial products

Exercise 3: (partial product length) **(8 points)**
We define a partial product

$$S'_{2j,2k} = \sum_{t=j}^{j+k-1} \langle g_{2t} \rangle \cdot 4^{t-1}$$

Prove for all k that $S'_{2j,2k}$ is a multiple of 2^{2j-2} bounded by $S'_{2j,2k} \leq 2^{n+2j+2k+2}$.

Exercise 4: (number of excess full adders) **(10 points)**
Prove or disprove: the number of excess full adders in binary addition tree for Booth recoding is $O(m \log m)$.