University of the Saarland
Department 6.2 - Informatik
Prof. Dr. W.J. Paul

Computer Architecture I - WS 03/04

(due: 5.11.2003)

Today is the last day when you can be registered for this course. If you will not be registered your solutions will not be graded and you cannot take part in the final exam.

The registration form is available in **http://www-wjp.cs.uni-sb.de/lehre/lehre.php** under "Computer Architecture".

Excercise 1: (partition lemma) **(4 points)**

Consider a number representation to base B, where string $a \in \{0, \dots, B-1\}^n$. For string a we denote by

$$\langle a \rangle_B = \sum_{i=0}^{n-1} a_i * B^i, \text{ where } a_i \in \{0, ..., B-1\}$$

the natural number that has a as its representation to base B.

Prove that for any $m \in \{0, \dots, n-1\}$ holds:

$$\langle a[n-1:0] \rangle_B = \langle a[n-1:m] \rangle_B * B^m + \langle a[m-1:0] \rangle_B$$

Excercise 2: (addition algorithm for numbers to base B) **(4 points)**

Addition algorithm for numbers to base B (represented in the previous exercise) is the following:

$$\begin{aligned} c_{in} &= c_{-1} \\ \langle c_i s_i \rangle_B &= a_i + b_i + c_{i-1} \\ s_n &= c_{n-1} \end{aligned}$$

Prove that

$$\langle c_{n-1} s[n-1:0] \rangle_B = \langle a[n-1:0] \rangle_B + \langle b[n-1:0] \rangle_B + c_{in}$$

Excercise 3: (addition algorithm for 1-digit decimal numbers) **(5 points)**

We can represent natural numbers by binary representations or by unary representations.
What ARE numbers in $\mathbb{N}$ or $\mathbb{N}_0$?
Assume $\mathbb{N}$ consists of sequences of bars $|$, i.e. $\mathbb{N} = \{|\}^+$ and 0 denotes the empty string ε.

$$\begin{aligned} \text{Thus } \langle 5 \rangle_{10} &= ||||| \\ \langle 10 \rangle_{10} &= |||||||||| \\ \langle 10 \rangle_2 &= || \\ \langle 0 \rangle_{10} = \langle 0 \rangle_2 &= \varepsilon \end{aligned}$$

Then for strings $x, y \in \mathbb{N}_0$, addition is just concatenation:

$$x + y = xy$$

For example: $|| + ||| = |||||,\ || + \varepsilon = ||$

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We need to construct a table for addition algorithm for base $B = 10$ like that:

a_i	b_i	c_{i-1}	c_i	s_i
$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$
4	1	0	0	5
$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$
5	6	1	1	2
$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$

Prove (using unary numbers) that the following statements indeed hold:

$$a)\ 5 + 6 + 1 = \langle 1, 2 \rangle_{10}$$
$$b)\ 4 + 1 + 0 = \langle 0, 5 \rangle_{10}$$

Exercise 4: (associativity) **(3+4 points)**

Show that the following operations are associative:

$$a)\ \wedge : \{0,1\}^2 \longrightarrow \{0,1\}$$
$$b)\ \circ : \{\{0,1\}^2\}^2 \longrightarrow \{0,1\}^2$$

where $\circ$ defined by

$$\begin{aligned}(g,p) &= (g_2,p_2) \circ (g_1,p_1) \\ &= (g_2 \vee g_1 \wedge p_2, p_1 \wedge p_2)\end{aligned}$$