Saarland University
Department 6.2 - Computer Science
Prof. Dr. W. J. Paul
Dr. Mikhail Kovalev

Theoretical Computer Science - WS13/14

Exercise Sheet 2 (due: 26.02.14, 48 points)

Exercise 1: (regular languages) (5 + 5 points)

Prove the following lemma:

$$L \text{ is regular } \iff \exists \text{ regular grammar G} : L = L(G)$$

Exercise 2: (context free languages) (3 + 3 points)

Let $G_1 = (T_1, N_1, P_1, S_1)$ and $G_2 = (T_2, N_2, P_2, S_2)$ be context free grammars with $N_1 \cap N_2 = \emptyset$. Show that the following languages are also context free:

1. $L(G_1) \cup L(G_2)$,
2. $L(G_1) \cdot L(G_2)$

Exercise 3: (context free languages) (8 points)

Let L be a context free language and L' be a regular language. Show that $L \cap L'$ is a context free language.

Exercise 4: (pushdown automata) (5 + 5 points)

Construct non-deterministic push-down automata $M = (\Sigma_I, \Sigma_p, Z, \delta, z_0, Z_a)$ which accept the following languages:

1. $L = \{a^n b^n \mid n \in \mathbb{N}_0\}$
2. $L = \{a^m b^n \mid m \leq n \leq 2m\}$

Exercise 5: (context free grammars) (3 + 3 + 3 points)

Construct context free grammars, which accept the following languages:

a) $\{wcw^R \mid w \in \{a,b\}^*\}$ b) $\{ww^R \mid w \in \{a,b\}^*\}$ c) $\{a^n b^n c^m \mid n, m \in \mathbb{N}_0\}$

Notation: w^R denotes the inverse of w. For $w = w_1 \cdots w_n$ we have $w^R = w_n \cdots w_1$.

Exercise 6: (pumping lemma) (5 points)

Prove that language $L = \{a^n b^n c^n \mid n \geq 0\}$ is not context free.