

Flex Ray: Serial Interface - a Formal Model for Coding and Decoding

Seminar: The FlexRay Communication Protocol

Chair of Prof. Dr. W. J. Paul

Talk by Michael Gerke

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Overview

- General remarks
- Encoding
- Low level bit transfer
- Decoding:
 - Voting
 - Strobing

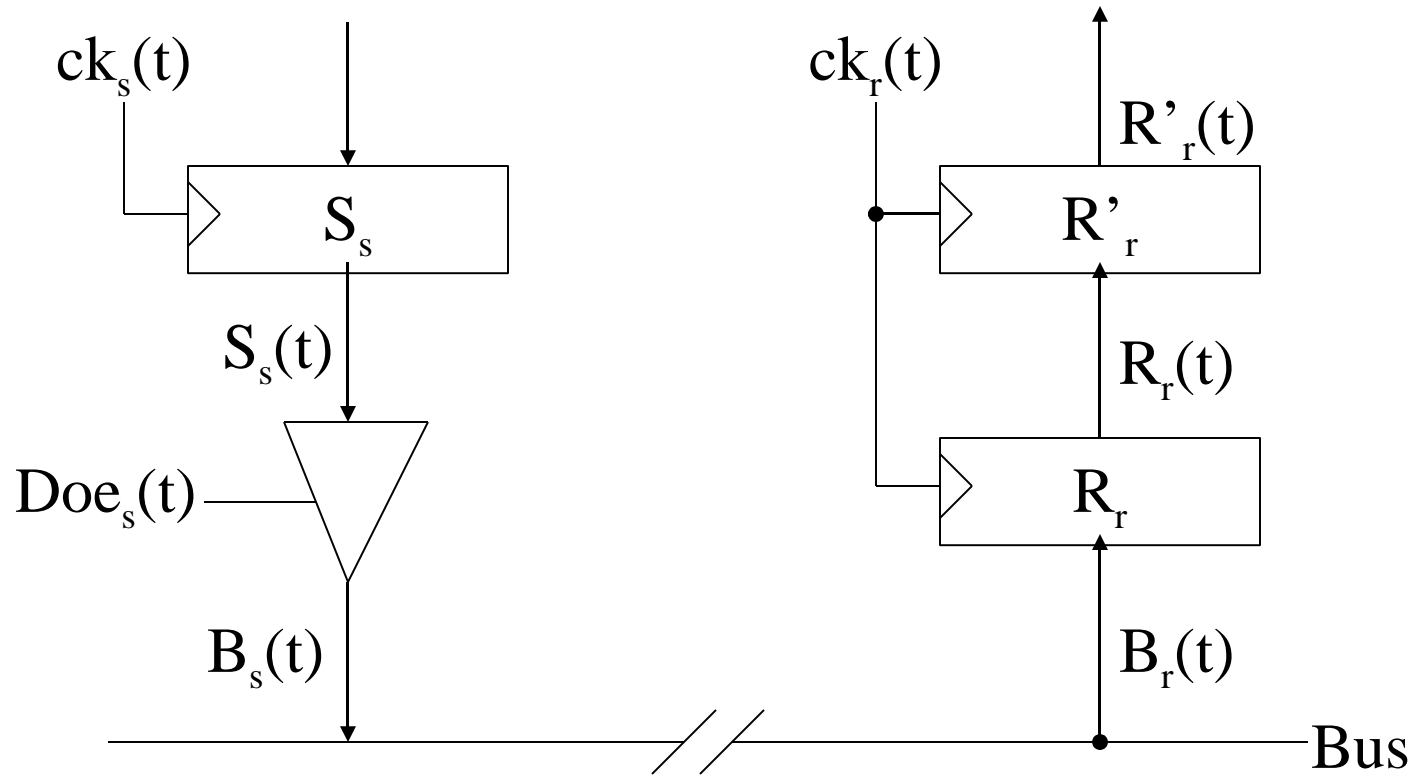
General remarks

- TSS = 0¹
- We ignore glitches
- Reception controlled by different state machine

Definitions: Clocks

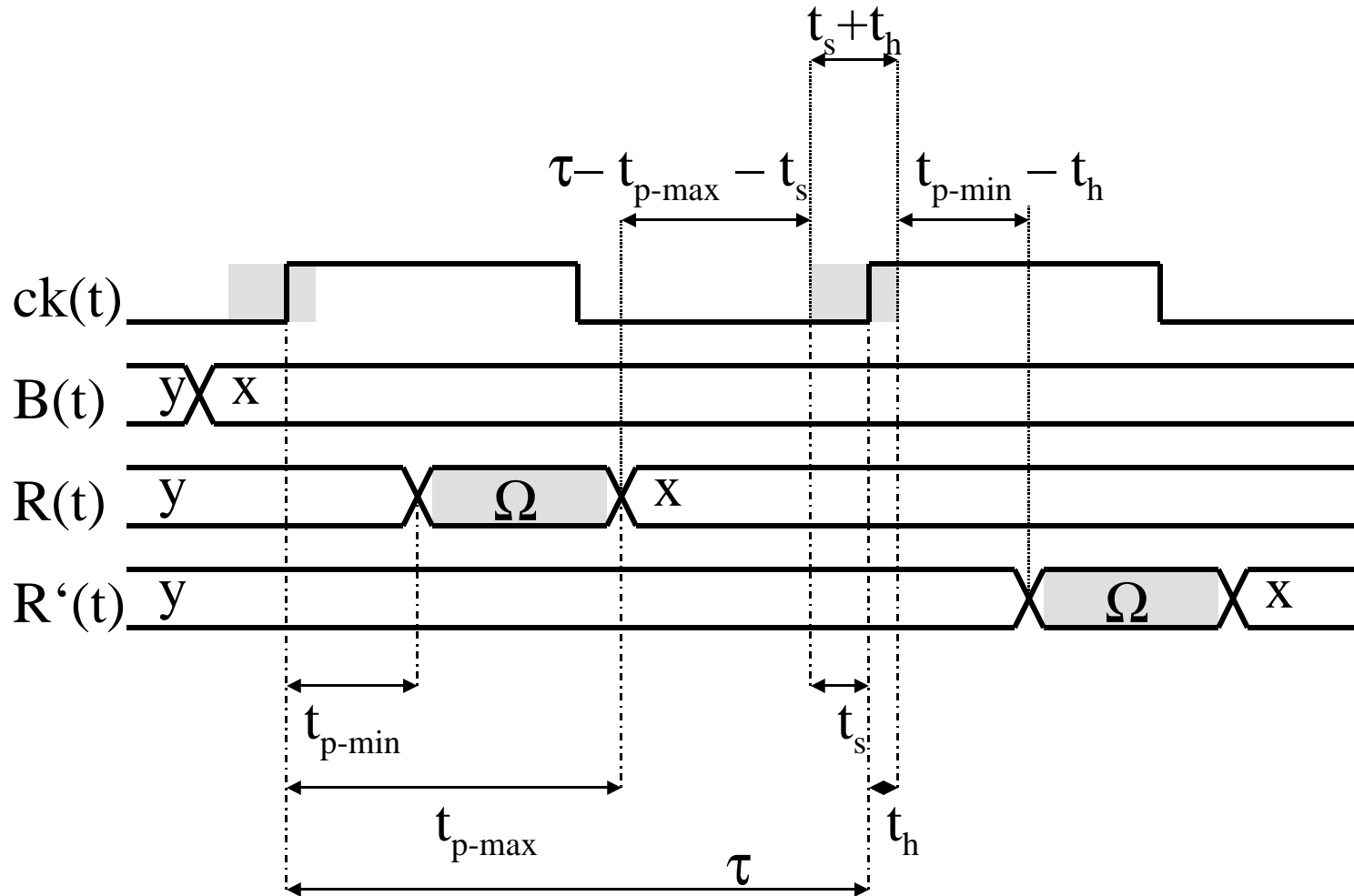
- Node number u : ECU_u has clock signal $ck_u(t)$ with cycle time T_u
- We assume drift is at most 0.15%
- $e_u(i)$: i^{th} rising edge of ck_u
- i^{th} cycle of ECU_u : $[e_u(i), e_u(i+1)[$

Serial bus interface:



Lower indices: X_s =sender's X and X_r =receiver's X

Definition: Register semantics

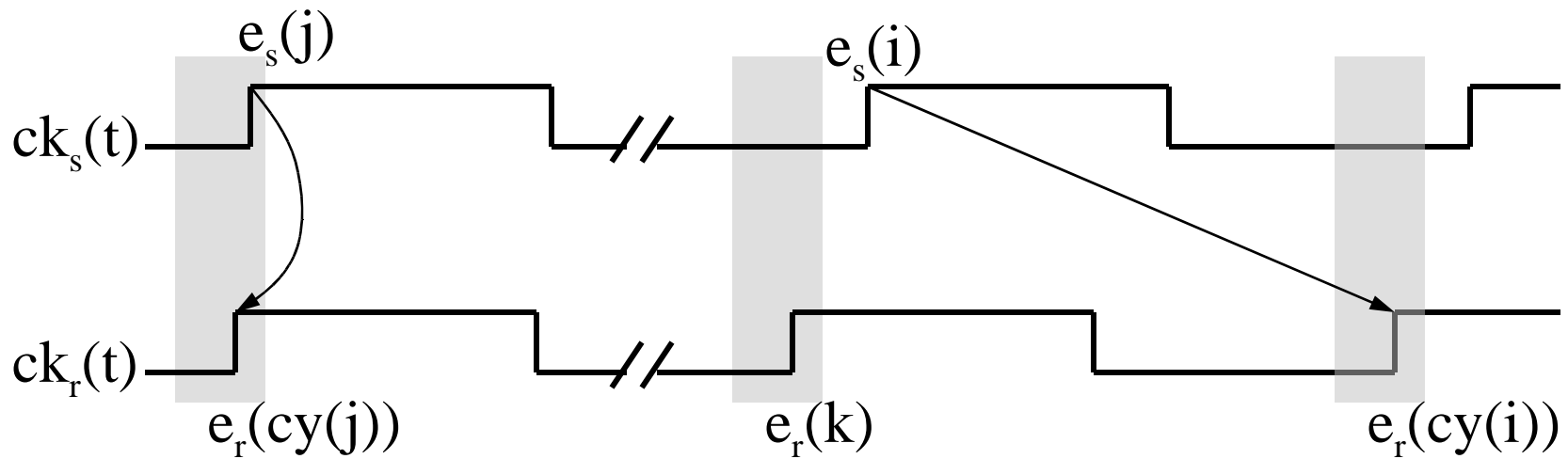


Definition: Formal register semantics

- Old value of $R=y$.
- $B(t)=x: t \in [e(i)-t_s, e(i)+t_h]$: sampling interval
- $R(t) = \begin{cases} y & : t \leq e(i) + t_{p-\min} \\ & : e(i) + t_{p-\min} < t < e(i) + t_{p-\max} \\ x & : t \geq e(i) + t_{p-\max} \end{cases}$
- $R'(t) \in \{0,1\}$
- normal: $R'(e(i)+t_{p-\max})=R(e(i)-t_s)$

Note that we get a delay of 1 caused by the second register R' .

Definition: Bus connection



First affected receiver cycle:

$$\text{cy}(i) = \max \{ k | e_r(k) + t_h < e_s(i) \} + 1$$

Definition:

Formal Bus connection

If the sender s puts new value B_s^i on the bus in cycle i at clock edge $e_s(i)$:

$$B_s^i \neq B_s^{i-1}$$

The first affected receiver cycle is denoted by:

$$cy(i) = \max \{ k | e_r(k) + t_h < e_s(i) \} + 1$$

*Upper indices: $X^i = X$ directly before the end of cycle i
(when all hardware has stabilized)*

Lemma 1

IF $x = B_s^i = \dots = B_s^{i+7}$

THEN $R_r^{cy(i)+k+1} = R_r^{cy(i)+k} = x; k \in [\beta : \beta + 6]; \beta \in \{0, 1\}$

If the sender holds the bus stable for eight consecutive cycles, then the receiver samples during at least 7 consecutive cycles the correct value x . The value of β depends on the difference between sender and receiver clock and is either 0 or 1.

Proof: Lemma 1

Let clock drift be bounded by 0.15% and only one node be sending.

The sampling intervals of all receiver edges $c_{y(i)+k}$ are in a region of time where the bus is stable.

If the sampling interval for $k=0$ is not in this region, then the sampling interval for $k=7$ is and vice versa, so I can select $\beta \in \{0,1\}$ such that the Lemma holds.

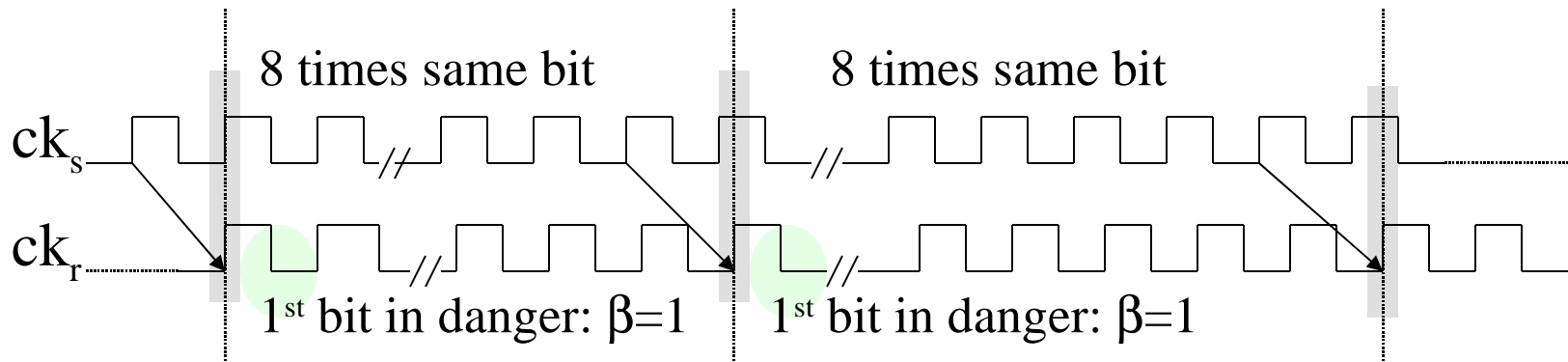
Lemma 2

IF $x = B_s^{i-8} = \dots = B_s^{i-1}$ and $\neg x = B_s^i = \dots = B_s^{i+7}$

THEN for i' : $\neg x = R_r^{i'} \neq R_r^{i'-1}$: $i' \in \text{cy}(i) + [0:1] + 1$

If the sender transmits x in cycles $i-8$ to $i-1$ and $\neg x$ in cycles i to $i+7$, then the cycle i' in which $\neg x$ occurs for the first time in is bounded by an interval of two cycles: $i' \in \text{cy}(i) + [0:1] + 1$

Proof: Lemma 2



As clock drift is bounded by 0.15%, we know that for two succeeding intervals of 8 consecutively sent bits the value of β is the same.

Lemma 3

$$\forall i: \forall k < 600: cy(i+k) \in cy(i) + k + [-1:1]$$

During 600 cycles, a clock can get at most one cycle difference to the idealized clock due to drift.

Proof: Lemma 3

Usually: $cy(i+1)=cy(i)+1$, clock drift can cause:
 $cy(i+1)=cy(i)$ or $cy(i+1)=cy(i+2)$

As drift is bounded by 0.15%, this can happen at most once in $1/0.0015 > 600$ cycles.

Definition: Frame assembly

m : message to be transferred
↓

$f(m)$: frame to be sent (and to be reassembled)
↓

$F(m)$: bit vector to be transmitted

Definition: Frame assembly

$f(m)=$

TSS FSS BSS $m[0]$... BSS $m[l-1]$
FES

As each bit is transmitted for 8 cycles:

$F(m)=f(m)[0]^8 \dots f(m)[l-1]^8$

Sender cycles are numbered such that:

$B_s^i = F(m)[i]$

Lemma 4

$\forall f(m)[i]: \exists \beta \in \{0, 1\}: \forall k \in [\beta: \beta+6]:$

$$R_r^{cy(8 \cdot i) + k + 1} = R_r^{cy(8 \cdot i) + k} = f(m)[i]$$

This means the bit $f(m)[i]$ is correctly sampled at receiver edge $cy(8 \cdot i) + k$

Proof: Lemma 4

Bus stable for 8 consecutive cycles:

$$B_s^{8 \cdot i + k} = f(m)[i] \text{ for } k \in [0:7]$$

Apply Lemma 1

Voting: Definition

v^j =majority vote over last five R' values:

R'^j, \dots, R'^{j-4}

Note that we get a delay of 2 cycles caused by the voting process.

Lemma 5

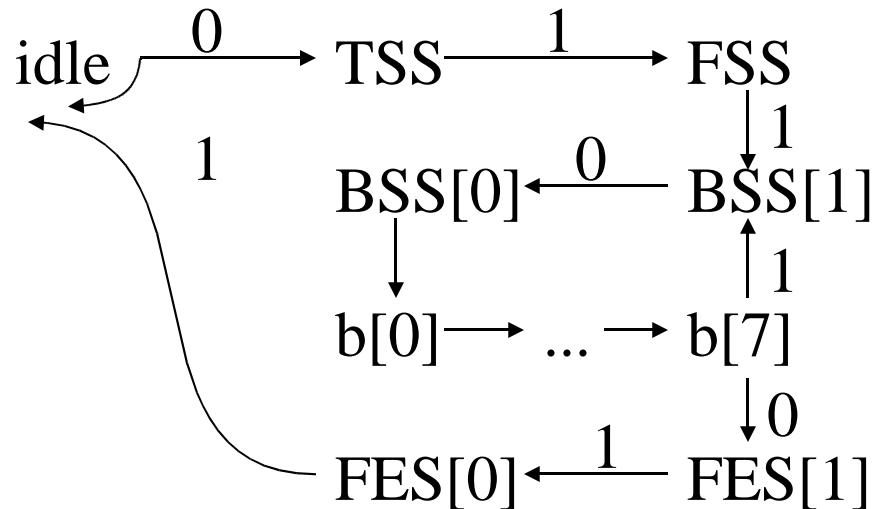
$$\forall f(m)[i]: \exists \beta \in \{0, 1\}: \forall k \in [\beta+2:\beta+8]: \\ v^{\text{cy}(8 \cdot i)+k+1} = f(m)[i]$$

This means the bit $f(m)[i]$ is correctly voted at receiver edge $\text{cy}(8 \cdot i)+k+1$

Proof: Lemma 5

Lemma 4 entails that in cycles $cy(8 \cdot i) + k + 1$ for $k \in [\beta + 2 : \beta + 8]$ we have received at least three copies of bit $f(m)[i]$.

Bit strobing: Automaton



Transition function: $\Delta(s,i)$

Automaton clocked at: strobe^t

Bit strobing: Definitions

strobe point: $\text{strobe}^j = (\text{cnt}^j = 4)$

$$\text{state}^{t+1} = \begin{cases} (\text{state}^t, v^t) : \text{strobe}^t \\ \text{state}^t & : \text{otherwise} \end{cases}$$

$\text{sync}^j =$

$$((\text{state}^j = \text{idle}) \wedge v^{j-1}) \vee ((\text{state}^j = \text{BSS}[1]) \wedge v^{j-1} \wedge \neg v^j)$$

$$\text{cnt}^{j+1} = \begin{cases} 1 & : \text{sync}^j \\ (\text{cnt}^j + 1) \bmod 8 & : \text{otherwise} \end{cases}$$

Bit strobing: Definitions

$\text{str}(h)$ denotes the index of the cycle of the $(h+1)^{\text{th}}$ activation of the strobe signal

$\text{sy}(h)$ denotes the index of the (last) cycle of the $(h+1)^{\text{th}}$ activation of the sync signal

$\text{nb}(h)$ is the number of bits of $f(m)$ sent in synchronization interval $[\text{sy}(h):\text{sy}(h+1)]$

$$\text{NB}(h) = \sum_{h' < h} \text{nb}(h')$$

The Theorem: Motivation

We want to show that the message is correctly reassembled by the receiver.

In order to do so, we will show that the automaton and the syncing work as expected and thus the right bits are strobed.

These criteria will be formulated as an invariant.

Invariant

- 1) Automaton correctly monitors the received bits
- 2) Message bits are correctly strobed
- 3) Transitions of automaton occur fast enough,
i.e. before the next bit can be sampled
- 4) Sync signals are activated at expected times
- 5) Strobe signals are activated at expected times

Lemma 7 Preconditions

For any receiver cycle j ,

for any $k = \text{NB}(h') + k'$ with

$\text{str}(k) \leq j$ and

$k' \in [0 : \text{nb}(h') - 1]$,

and for any h with

$\text{sy}(h) \leq j$

it holds:

Lemma 7 Preconditions

For any receiver cycle j ; *Induction over j*

for any $k = \text{NB}(h') + k'$ with

$\text{str}(k) \leq j$ and

$k' \in [0: \text{nb}(h') - 1]$,

and for any h with

$\text{sy}(h) \leq j$

it holds:

Actual sync number: h

$$\text{NB}(h') \leq k \leq \text{NB}(h)$$

*Number of actual bit
in this sync interval*

*Number of bits sent in previous
sync intervals $0, \dots, h'$*

*Number of bits to be sent
in this sync interval*

Lemma 7 Part 1

**1) If strobe k is the last strobe before cycle j ,
i.e. $j \in [\text{str}(k)+1:\text{str}(k+1)]$, then state^j is given
as expected (see Automaton):**

In the first sync interval ($h'=0$) state^j is equal to:

TSS for $k'=0$; FSS for $k'=1$ or BSS[1] for $k'=2$

In the other sync intervals

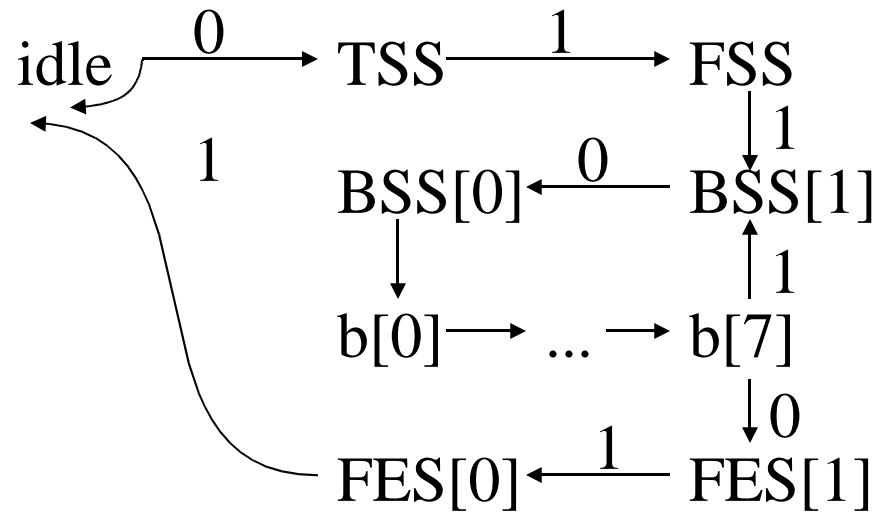
(of length $\text{nb}(h') \in [10:11]$), state^j is equal to:

BSS[0] for $k'=0$ or $b[k'-1]$ for $k' \in [1:8]$

All but the last sync interval ($h' < l$) end with $\text{state}^j = \text{BSS}[1]$
for $k'=9$

For $h'=l$ we have $\text{state}^j = \text{FES}[10-k']$ for $k' \in [9:10]$

Reminder: Automaton



Lemma 7 Parts 2-5

2) The sampled signals satisfy $v^{\text{str}(k)} = f(m)[k]$

3) $\text{str}(k) + 1 < \text{cy}(8 \cdot (k + 1)) + [2:3] + 1$

4) $\text{sy}(h) \in \text{cy}(8 \cdot \text{NB}(h)) + [2:3] + 1$

5) $\text{str}(k) = \text{sy}(h') + 8 \cdot (k - \text{NB}(h')) + 4$

Lemma 7 Proof Plan

We do an induction over j .

$$I4(j) \wedge I5(j) \Rightarrow I2(j+1) \wedge I3(j+1) \quad (\text{sub-lemma: lemma 6})$$

$$I2(j+1) \wedge I3(j+1) \Rightarrow I1(j+1) \quad (\text{trivial})$$

$$I1(j+1) \wedge I3(j+1) \Rightarrow I4(j+1) \wedge I5(j+1)$$

$$I4(j) \wedge I5(j) \Rightarrow I2(j+1) \wedge I3(j+1)$$

We want to show:

2) *The message bits are correctly strobed:*

The sampled signals satisfy $v^{\text{str}(k)} = f(m)[k]$

3) *Transitions of the automaton occur fast enough, i.e. before the next bit can be sampled:*

$$\text{str}(k) + 1 < \text{cy}(8 \cdot (k+1)) + [2:3] + 1$$

Lemma 6

Assuming that sender cycles $NB(h)$ and corresponding receiver cycles are not too far apart:

IF

(1) *Strobe point occurs in the expected time bounds and if*

(2) *Syncing occurs in the expected time bounds*

THEN

(i) *The message bits are correctly strobed*

(ii) *Transitions of the automaton occur fast enough, i.e. before the next bit can be sampled*

Lemma 6

IF h' maximal such that

(1) $\text{str}(k) = \text{sy}(h') + 8 \cdot (k - \text{NB}(h')) + 4$ and if

(2) $\text{sy}(h') \in \text{cy}(8 \cdot \text{NB}(h')) + [2:3] + 1$

THEN

(i) $v^{\text{str}(k)} = f(m)[k]$ and

(ii) $\text{str}(k) + 1 < \text{cy}(8 \cdot (k + 1)) + [2:3] + 1$

Proof: Lemma 6 (i)

Part(i) using Lemma 3 and Lemma 5:

$$\begin{aligned} \text{str}(k) &= \text{sy}(h') + 8 \cdot (k - \text{NB}(h')) + 4 \\ &\in \text{cy}(8 \cdot \text{NB}(h')) + 8 \cdot (k - \text{NB}(h')) + [6:7] + 1 \\ &\in \text{cy}(8 \cdot (\text{NB}(h') + k - \text{NB}(h'))) + [5:8] + 1 \end{aligned}$$

$$v^{\text{str}(k)} = f(m)[k]$$

Proof: Lemma 6 (ii)

Part(ii) using Lemma 3:

$$\begin{aligned} \text{str}(k)+1 &\in \text{cy}(8 \cdot \text{NB}(h')) + 8 \cdot (k - \text{NB}(h')) + [6:7] + 1 + 1 \\ &= \text{cy}(8 \cdot \text{NB}(h')) + 8 \cdot (k - \text{NB}(h') + 1) + [0:1] \\ &\in \text{cy}(8 \cdot (\text{NB}(h') + k - \text{NB}(h') + 1)) + [-1:2] \\ &< \text{cy}(8 \cdot (k+1)) + [2:3] + 1 \end{aligned}$$

$$I1(j+1) \wedge I3(j+1) \Rightarrow I4(j+1)$$

We want to show:

4) *sync signals are activated at expected times:*

$$sy(h) \in cy(8 \cdot NB(h)) + [2:3] + 1$$

Lemma 7 Proof Part 4

We have to show:

- (iii) The falling edge that triggers $sy(h)$ is seen by the receiver during the right cycle j
- (ii) The automaton is in the state $BSS[1]$ during cycle j

Lemma 7 Proof Part 4(i)

Lemma 2 combined with Lemmas 4 and 5 shows that the falling edge which triggers $sy(h)$ is seen in v^j for $j \in cy(8 \cdot NB(h)) + [2:3] + 1$

From 2:

First seen in n' : $n' \in cy(n) + [0:1] + 1$

From 4,5:

$f(m)[i] = v^{cy(8 \cdot i) + [2:3] + 1}$

Lemma 7 Proof Part 4(ii)

Part1 implies $\text{state}^j = \text{BSS}[1]$ for cycles $j \in [\text{str}(k)+1:\text{str}(k+1)]$

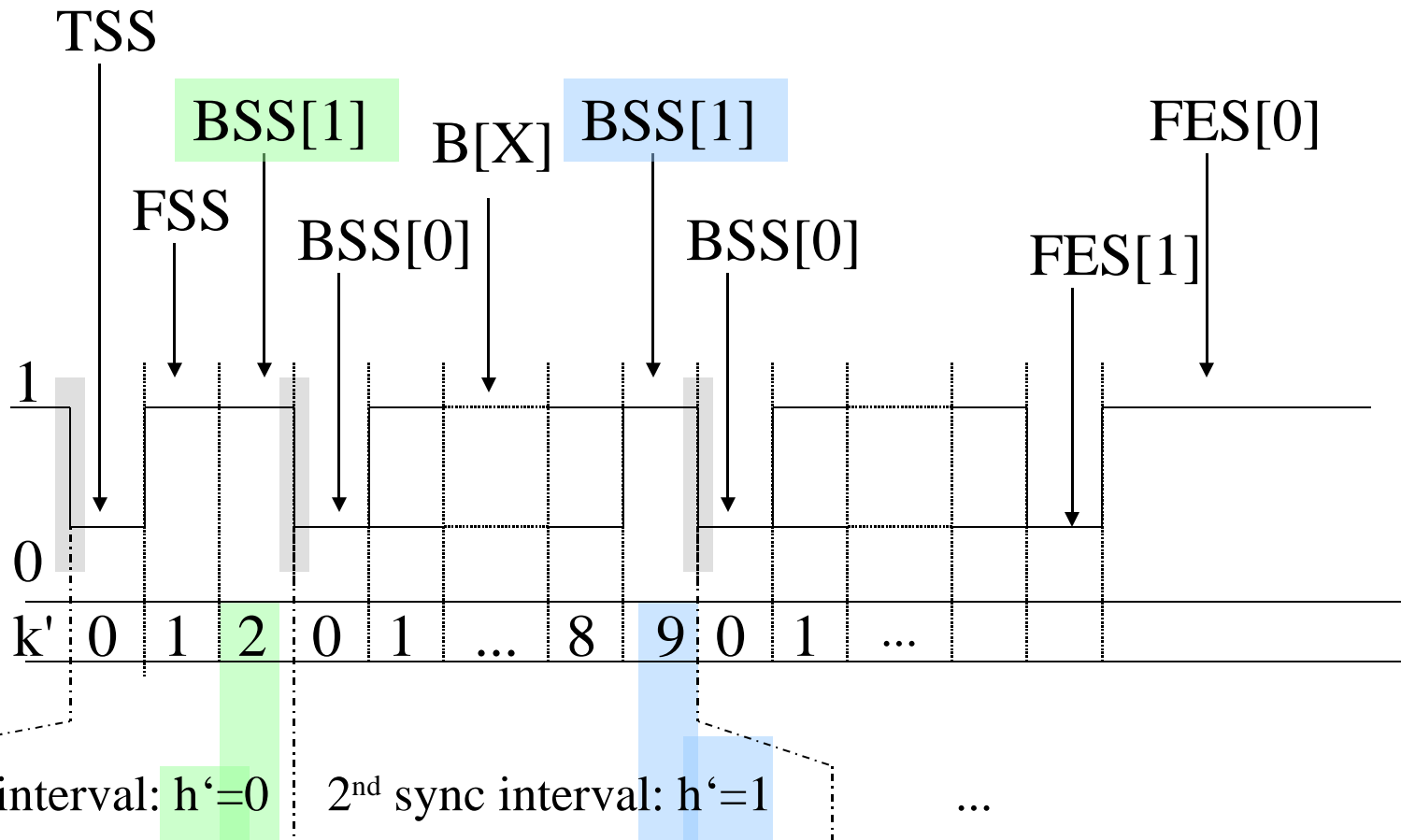
if k is maximal and

$$(\text{h}'=0 \wedge \text{k}'=2) \vee (\text{h}' \in [1:l-1] \wedge \text{k}'=8).$$

Outside these time intervals the sync signal cannot become active.

Encoding of Frames

remember: $k = NB(h') + k'$



Lemma 7 Proof Part 4(ii)

Part 3 implies

$$\text{str}(k)+1 < \text{cy}(8 \cdot (\text{NB}(h') + k' + 1)) + [2:3] + 1$$

Thus the automaton is in state BSS[1] one cycle before the first zero of the BSS[0] bit can be possibly sampled.

$$\mathbf{I1(j+1) \wedge I3(j+1) \Rightarrow I5(j+1)}$$

We want to show:

5) strobe signals are activated at expected times:

$$\text{str}(k) = \text{sy}(h') + 8 \cdot (k - \text{NB}(h')) + 4$$

Lemma 7 Proof Part 5

For the case $k'=0$:

$$\begin{aligned} \text{sy}(h') &\in \text{sy}(h'-1)+8\cdot\text{nb}(h'-1)+[-1:1] \\ &= \text{sy}(h'-1)+8\cdot(\text{nb}(h'-1)-1)+8+[-1:1] \end{aligned}$$

From the induction hypothesis:

$$\begin{aligned} \text{str}(k-1) &= \text{sy}(h'-1)+8\cdot(k-1-\text{NB}(h'-1))+4 \\ &= \text{sy}(h'-1)+8\cdot(\text{nb}(h'-1)-1)+4 \end{aligned}$$

Thus $\text{str}(k-1)$ is before $\text{sy}(h')$ and there is no additional strobe between them.

Lemma 7 Proof Part 5

For the case $k' > 0$ part 5 follows from the induction hypotheses. ($k = \text{NB}(h') + k'$)

Definition: Frame reassembly

After reset: empty reconstruction frame f'^0

$$f'^{t+1} = \begin{cases} f'^t \circ v^t & : \text{strobe}^t \\ f'^t & : \text{otherwise} \end{cases}$$

The theorem

Let clock drift $\delta \leq 0.0015$

Let $L = 8 \cdot l'$ be the length of $F(m)$

$$f^{\lceil (1+\delta) \cdot L \rceil + 8} = f(m)$$