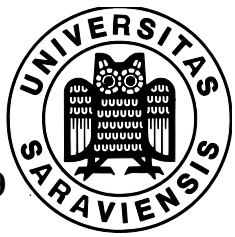
Saarland University
Department 6.2 - Computer Science
Prof. Dr. W. J. Paul
M. Sc. Andrey Shadrin



Computer Architecture II - WS 08/09

Exercise Sheet 6 (due: 8.12.08)

Excercise 1: (Shifter Properties) (5 points)
Let $a \in \{0,1\}^n$ and $i \in \{0,..,n-1\}$. Prove the following shift properties:

- $\langle lls(a,i) \rangle \equiv \langle a \rangle 2^i \bmod 2^n$
- $\langle lrs(a,i) \rangle \equiv \lfloor \langle a \rangle / 2^i \rfloor$
- $[ars(a,i)] \equiv \lfloor [a] / 2^i \rfloor$

where

- lls - logical left shift,
- lrs - logical right shift,
- ars - arithmetic right shift.

Excercise 2: (Convergence of Newton-Raphson) (3 points)
In the convergence proof of the Newton-Raphson iteration we divided the approximation error δ into three parts:

- $\delta_1 = 1/fb - (x_i(2 - f_b x_i))$
- $\delta_2 = x_i(2 - f_b x_i) - x_i A_i$
- $\delta_3 = \lfloor x_i A_i - xiAi \rfloor_\sigma$

Explain the cause for each δ_i.

Excercise 3: Binary Fractions (2 points)
Let $f = f[0].f'[1:p-1]$.
Show that $f[0] \leq f \leq f[0] + \langle f'[1:p-1] \rangle 2^{-(p-1)}$.

Excercise 4: Encoder (10 points)
An encoder is a circuit, which transforms a unary number into its binary representation. Formaly, $a[\langle enc(a) \rangle] = 1$, where $enc : \{0,1\}^m \to \{0,1\}^{log_2(m)}$, unary $a[n-1:0] = (0^{n-i-1}, 1, 0^i)$ for some $i \in \{0,...,n-1\}$ and m are power of two. Construct this circuit and prove its correctness.