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Computer Architecture I – WS 06/07

Exercise Sheet 3 **(due: 13.11.06)**

Exercise 1: (Wallace Tree; Exact Costs) **(5 points)**

In class, we have seen Wallace trees constructed as balanced 'binary' trees of 4/2 adders. Compute the exact cost of such a tree for the case that $m = 2^k$.

Exercise 2: (Wallace Tree; Optimization) **(5 points)**

The 4/2 adder presented in class has a delay of two full adders.

1. Analyse the output paths and try to optimize the delay of your construction.
2. Prove that your construction is still correct.

Exercise 3: (Wallace Tree) **(8 points)**

Construct a Wallace tree from 3/2 adders only, i.e., construct a balanced tree of 3/2 adders (this is *not* a binary tree). Specify for each node of that tree the partial sum $S_{i,k}$ that it computes. Compute the delay of your construction and compare the result with the delay of the construction from class.

Exercise 4: (Optimization) **(5 points)**

In our delay model, some inverting gates (*nand* and *nor*) are faster than their non-inverting counterparts.

1. Use this fact to construct a delay-optimized *or*-tree, which computes for an input $a \in \mathbb{B}^n$ for $n = 2^k$ the output
$$b = a[n-1] \vee \cdots \vee a[1] \vee a[0] \ .$$
2. Compute the delay and cost of your construction as a closed formula.
3. Prove the correctness of your construction rigorously.

Exercise 5: (Binary Comparison) **(3+2+2+2=9 points)**

Within the ALU construction in the lecture notes, the binary comparison for two two's complement values $a, b \in \mathbb{B}^n$ defined as

$$[a] \circ [b] \text{ for } \circ \in \{<, >, \leq, \geq, =, \neq\}$$

is implemented by a subtractor and an equality tester.

In this exercise you are asked to construct an different, special circuit for this binary comparison applied to two binary values. Hence, you should construct a circuit that calculates

$$\langle a \rangle \circ \langle b \rangle \text{ for } \circ \in \{<, >, \leq, \geq, =, \neq\}$$

Since your construction should be a special purpose circuit, do not use the construction from the lecture notes in the following.

1. Construct a circuit that computes the output $c \in \mathbb{B}$ with $c = (\langle a \rangle < \langle b \rangle)$ from the inputs a, $b \in \mathbb{B}^n$.
2. Extend your construction from 1. to compute the six comparisons from above.
3. Compute the delay and cost of your construction as a closed formula and compare those with the construction in the lecture.
4. The ALU constructions from the lecture and the MP00 book differ in the following way: Instead of using an equality tester directly on inputs a and b, a zero tester is used which checks against the output $s[n-1:0]$ being 0^{32}.

   Proof or falsify: If $sub = 1$, then $a = b \Leftrightarrow s = 0^{32}$.

   Note: $[a] - [b] \in T_n$ may *not* be assumed.