University of the Saarland
Department 6.2 - Informatik
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Computer Architecture I - WS 02/03

(due: 18.11.2002)

Excercise 1: (Weights of nodes) **(8 points)**

In the lecture we defined the weight $W(\nu)$ of a node ν in the addition tree of multipliers by:

- For leaves: $W(\nu) = \begin{cases} 3 \text{ for } 3/2 \text{ adders} \\ 4 \text{ for } 4/2 \text{ adders} \end{cases}$
- For inner nodes with left son λ and right son ρ : $W(\nu) = W(\lambda) + W(\rho)$.

Show by induction on the level l in the addition tree:

1. In level l weights are nondecreasing from left to right.
2. The sum of weights of all nodes in level l is m.

Exercise 2: (Leading Zero Counter) **(8+4 points)**

A leading zero counter is a circuit which computes the number of leading zero of a given bit string $a[n-1:0]$.

In figure 1 you see two circuits which are helpful for computing the number of leading zeros of a. In figure 2 you see as an example a leading zero counter for 8 bit inputs that is composed by those two circuits. In a first step the input bits are feed into $n/2$ many LZero-a circuits and then there is a tree of LZero-b circuits which computes the result $b[4:0]$.

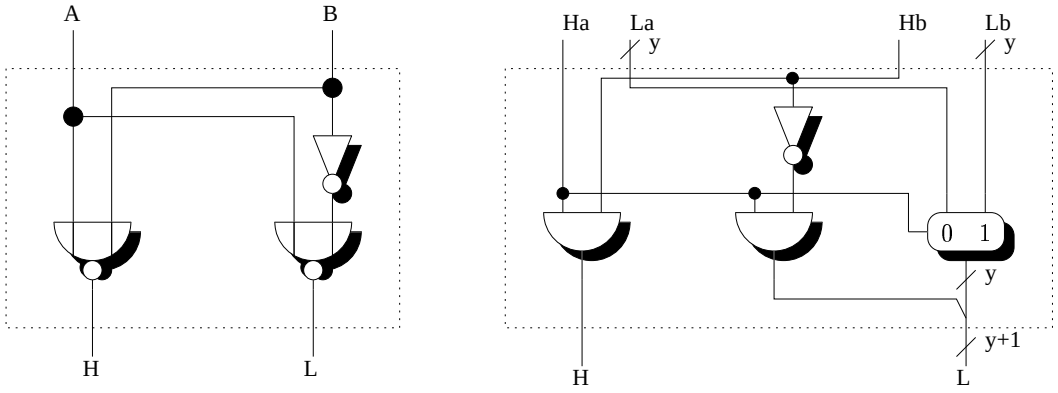

Figure 1: Components for leading zero counter

1. Show that circuits composed in this way compute correctly the number of leading zeros of their input(let n be a power of two).

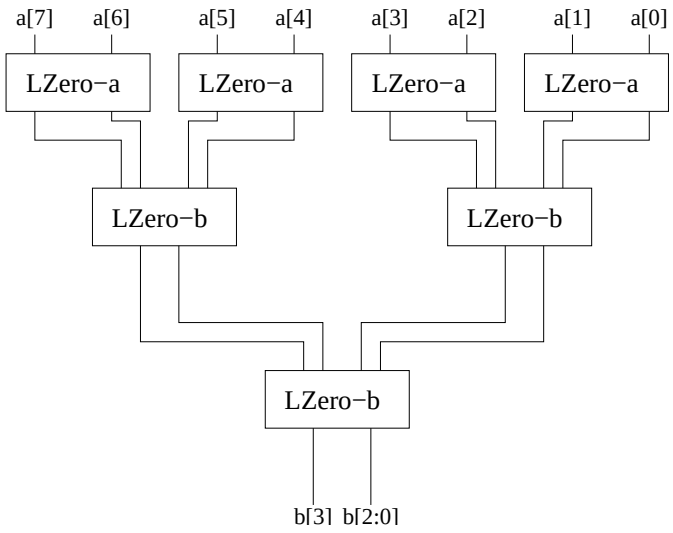

Figure 2: 8 bit leading zero counter

2. Derive a closed formula for the cost and delay of such a circuit. Take as cost for a multiplexer of width n $C(mux) = 3 * n + 1$ and as its delay $D(mux) = 3$. Prove the correctnes of your formulae.

Exercise 3: (Adder Tree) **(5+3+2 points)**

In the lecture we build the balanced addition tree of n,m multipliers with 4/2 adders (except the first level where we used 3/2 adders and 4/2 adders). One can build the addition tree directly with 3/2 adders.

1. A $m/2$ adder adds m input busses and outputs two output busses. Give a recursive construction of a m/2 adder consisting of 3/2 adders. Don't care about the bitwidth of the adders.

2. Derive a formula for the delay of your circuit.

3. For which m is your delay smaller then the delay of the tree from the lecture?