University of the Saarland
Department 6.2 - Informatik
Prof. Dr. W.J. Paul

Computer Architecture I - WS 02/03

(due: 04.11.2002)

Excercise 1: (cheap full adder) **(4 points)**

Find a cheapest full adder. You can use the following basic circuits to construct your full adder: AND, OR, INV, NAND, NOR, XOR, XNOR, MULTIPLEXER. All these basic circuits have cost one. The best possible full adder has cost three.

Excercise 2: (partition of bit vectors) **(4 points)**

Show that the following claim holds for any $j \in \{0, \ldots, n-1\}$:

$$\langle a[n-1:0] \rangle = \langle a[n-1:j] \rangle * 2^j + \langle a[j-1:0] \rangle$$

Exercise 3: (cost of parallel prefix computation) **(10 points)**

Derive a closed formula (without recursion or $\sum$ symbols) for the cost of the parallel prefix circuit from figure 1. For simplicity you can assume that $n = 2^l$ is a power of two. Proof the correctness of your formula by induction on l.

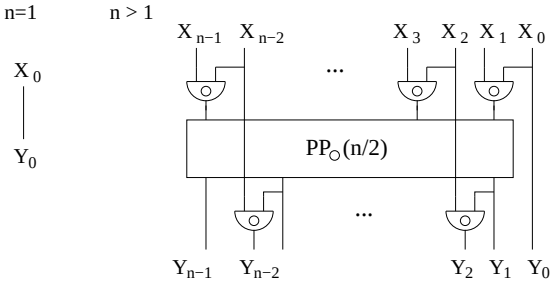

Figure 1: Parallel Prefix Circuit

Exercise 4: (Proof that $\circ$ is associative) **(12 points)**

Show that the operation $\circ$ defined by

$$\begin{aligned}(g,p) &= (g_2,p_2) \circ (g_1,p_1) \\ &= (g_2 \vee g_1 \wedge p_2, p_1 \wedge p_2)\end{aligned}$$

is associatove (see also figure 2)

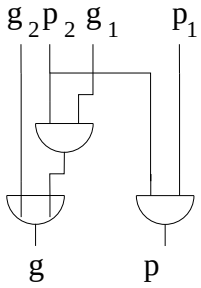

Figure 2: Construction of the associative $\circ$ operator