Saarland University
Department 6.2 - Computer Science
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Computer Architecture I - SS09

Exercise Sheet 1 (due: 11.05.09)

Organizational Notes:

- Please, register for the lecture on our web page. The registration is opened until 11.05.2009.
- Exercise sheets will be handed out on Mondays before the lecture. Your solutions will be collected one week after that (before the lecture).
- You are allowed to solve the exercises in groups of up to 3 students. Groups should not change over the semester. Everybody who has his name on a solution must be able to present it in tutorials.
- You need to solve 50% of all exercises in order to be admitted to the exam. In addition you must successfully present at least three solutions in tutorials.

Exercise 1: (two's complement numbers) (1 point)

In the lecture we introduced two's compliment numbers and their properties. For $a \in \{0,1\}^n$, $n \geq 2$ prove the following property:

$$[a] \equiv \langle a[n-2:0] \rangle \mod 2^{n-1}$$

Exercise 2: (cheap full adder) (3 point)

Design a cheapest full adder (single bit adder). The full adder takes three bits (a, b and carry-in c_{in} bits) as input and produces two signals (carry-out c_{out} and sum s bits) as output. You can use the following basic circuits to construct your full adder: AND, OR, INV, NAND, NOR, XOR, XNOR, MULTIPLEXER. Assume that all these basic circuits have cost one. The cheapest full adder design has cost three.

Exercise 3: (cost of parallel prefix computation) (5 + 3 point)

Derive a closed formula (without recursion or $\sum$ symbols) for the cost of the n-input parallel prefix circuit based on $\circ$-gates (considered in the lecture). For simplicity you can assume that $n = 2^l$ is a power of two. Prove the correctness of your formula by induction on l.

Exercise 4: (prove that $\circ$ is associative) (4 point)

Prove that the operation $\circ : \{\{0,1\}^2\}^2 \to \{0,1\}^2$ defined as

$$\begin{aligned}(g,p) &= (g_2,p_2) \circ (g_1,p_1) \\ &= (g_2 \vee g_1 \wedge p_2, p_1 \wedge p_2)\end{aligned}$$

is associative.