# Saarland University
**Department 6.2 - Computer Science**
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## Theoretical Computer Science - WS13/14
**Exercise Sheet 8 (due: 24.03.14, 45 points)**

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### Organizational notes:

- This is the last exercise sheet!

### Exercise 1: ($O$ notation) (5 points)
Show that

$$\sum_{n=0}^{\infty} \frac{n}{2^n} = O(1).$$

### Exercise 2: (constructible functions) (5 + 5 points)
Prove the following statements.

1. Function $f(n) = n^2 \cdot \lceil \log n \rceil$ is time constructible.
2. Let $f : \mathbb{N}_0 \to \mathbb{N}_0$ and $g : \mathbb{N}_0 \to \mathbb{N}_0$. If both $f(n)$ and $g(n)$ are time constructible, then the functions $f(n) + g(n)$ and $\max(f(n), g(n))$ are also time constructible.

### Exercise 3: (Blum's theorem for space) (5 points)
In the proof of the Blum's speedup theorem for space we defined the function $f : \mathbb{N}_0 \to \mathbb{N}_0$ as

$$f(n) = \begin{cases} 0 & s(n) \text{ defined and } \phi_s(n) = 1 \\ 1 & \text{otherwise} \end{cases}$$

Prove that

$$\phi_i = f \to (\beta_i(n) \geq r_{n-i} \text{ faa } n)$$

### Exercise 4: (speedup function) (5 points)
Let $r : \mathbb{N}_0 \to \mathbb{N}_0$ be a total, computable function. Explain how one computes a total, space constructible function $r' : \mathbb{N}_0 \to \mathbb{N}_0$, which is greater or equal than $r$:

$$\forall i \in \mathbb{N}_0 : r'(i) \geq r(i).$$

### Exercise 5: (pebble game) (10 points)
In the lecture we have defined the rules of the so-called *pebble game*. Let $P(n)$ be the smallest number of pebbles which suffices to play the game successfully on any tree of size $n$ with degree 2. Proof that

$$P(n) \leq max\{1, \lceil log(n) \rceil\}$$

### Exercise 6: (complexity classes) (10 easy points)
Reproduce the proof from the lecture (in detail). For every total computable function $t : \mathbb{N}_0 \to \mathbb{N}_0$ it holds

$$DTAPE(t(n)) \subseteq DTIME(2^{O(t(n))})$$