

System Architecture (block course) - SS13  
Exercise Sheet 8 (due: 4.09.13) - 11 points

---

**Exercise 1:** (4)

Prove that  $l(c.prn[i]) \neq rSt \Rightarrow c.prn[i + 1] = succ(c.prn[i])$  is invariant w.r.t. function calling and while loop execution.

**Exercise 2:** (3)

Recall the Aho-Ullman Algorithm, which we use to evaluate complex expressions using the minimum number of registers.

- (a) (1 point) Derive the following expression from the  $C0$  production rules starting from  $A$ , then use the Aho-Ullman Algorithm to evaluate the expression tree, starting at the nodes that derive constant values:

$$((500 + 60) - (90 * 30)) + 9 + 10 + 11$$

- (b) (1 point) How many registers did you need?
- (c) (1 point) Prove that for a binary expression tree with  $2^k - 1$  nodes, only  $k$  registers are required to evaluate the full expression.

**Exercise 3:** (4)

Formally prove the correctness of the  $n$ -bit multiplier, i.e., that the output  $c \in \mathbb{B}^n$  is the product of  $a, b \in \mathbb{B}^n$  modulo  $2^{32}$ :

$$c = a *_{32} b$$