Saarland University
Department 6.2 - Computer Science
Prof. Dr. W. J. Paul
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System Architecture (block course) - SS13

Exercise Sheet 5 (due: 26.08.13) - 33 points

Organizational notes:

- For feedback on the difficulty of the sheet, please write down the amount of time spent on the exercise sheet (in hours, excluding bonus exercises). This number is irrelevant for your admission but helps us adjust the amount of exercises on each sheet.

Name, matr. nr., time spent: _____

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Exercise 1: (7)

Recall directed graphs (V, E) as introduced in the lecture.

(a) (1 point) Give a directed graph with 4 nodes and 6 edges.

(b) (1 point) Give two different graphical representations of that graph.

(c) (1 point) Give the mathematical definition of a tree.

(d) (1 point) Give a tree with 5 nodes and 4 edges.

(e) (1 point) Give two different graphical representations of that tree.

(f) (1 point) Recall the definition of tree regions from the lecture. Give a tree region of that tree including 4 nodes.

(g) (1 point) Give the mathematical definition of a labelled tree.

Exercise 2: (14)

Recall context-free grammars (N, T, S, P).

(a) (2 points) Give a context-free grammar G such that the language of G, $L(G)$, equals to the strings of digits representing the natural numbers, i.e., $L(G) = \{0, \ldots, 9\}*$.

(b) (2 points) Draw a derivation tree for your grammar deriving 1951.

(c) (1 point) Give the underlying labelled tree in mathematical notation.

(d) (2 points) Give a context-free grammar G_2 such that $L(G_2) = \{({}^n x)^n | n \in \mathbb{N} \wedge x \in L(G)\}$.

(e) Consider the following context-free grammar (N, T, S, P):

$$T = \{a, \ldots, z\}$$
$$N = \{L, X\}$$
$$S = \{L\}$$

With the following production rules:

$$L \to |X|X\ L\ X\ X|X\ X\ L$$
$$X \to a| \ldots |z$$

Use this grammar to create a derivation tree for *sslall*, then prove or refute:

i. (1 point) $\{\epsilon\}$ is a tree region.

ii. (1 point) $\{\epsilon, 0, 1, 2\}$ is a tree region.

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iii. (1 point) $\{\epsilon, 20, 21, 22, 23\}$ is a tree region.

iv. (1 point) $\{\epsilon, 0, 1\}$ is a tree region.

v. (1 point) $\{\epsilon, 2, 20, 21, 22, 23\}$ is a tree region.

vi. (1 point) $\{0, 1, 2, 20, 21, 22, 23\}$ is a tree region.

vii. (1 point) The grammar is ambigious.

Exercise 3: **(3)**

Recall the theorem that the grammar for arithmetic expressions is not ambigious, which was proven in four steps. Prove the third step of the theorem.

Exercise 4: **(5)**

Recall the grammar of the language $C0$ given in the lecture which fits into one page but still has a lot of expressive power.

(a) (3 points) Give a short program according to the grammar of $C0$, where the derivation tree has labels $\langle TyDs \rangle$, $\langle VaDs \rangle$ and $\langle FuDs \rangle$.

(b) (1 point) Explain why your program (at this point in the lecture, 20th of August) is a completely meaningless string, like "sslall".

(c) (1 point) Explain why we allow pointer type typedefs to reference types not defined so far.

Exercise 5: **(4)**

Recall the construction of the n-bit multiplier with depth logarithmic in n hinted at in the lecture, which uses n-bit 3/2-adders.

(a) (2 points) Give the recursive definition of the n-bit multiplier with logarithmic depth.

(b) (2 points) Extend the single core MIPS processor given in the lecture with the n-bit multiplier and add an R-type instruction *mul* that multiplies two registers.